A second difficulty with the lens beam waveguide arises from the losses due to reflection at the lens surfaces and to the nonzero loss tangent of the dielectric material from which the lenses are fabricated. For presently available materials these losses become excessively large for wavelengths less than a few millimeters. For the reflecting beam waveguide, the losses due to the correcting elements arise only from the finite conductivity of the reflecting surfaces. Even at wavelengths as short as 0.1 millimeter, the calculated conduction losses are only approximately 0.02 db per iteration for aluminum reflectors, making the reflecting beam waveguide usable well into the submillimeter wave region.

ACKNOWLEDGMENT

The authors acknowledge the interest and support of Prof. P. D. Coleman, Director of the Ultramicrowave Group at the University of Illinois, who originally suggested to one of the authors, W. H. Steier, that he study this problem. The authors also wish to express their appreciation for the interest of Dr. T. Li of the Bell Telephone Laboratories, Inc. in this problem.

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Analysis of a Differential Phase Shifter

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Summary—**This paper presents the theory and analysis of a ganged pair of "line stretcher" microwave phase shifters. The error analysis shows that some of the errors inherent in a single phase shifter of this type can be reduced through the use of a differential system; however, the magnitudes of other errors may more than offset the reduction. Graphical data are included to facilitate the rapid determination of the limit of error for any specified angle measurement.**

INTRODUCTION

AN ERROR analysis by Schafer and Beatty of the reflectometer type phase shifter proposed by Magid leads to the conclusion that the accuracy of the device is limited by errors in the determination of the guide wavelength and position of the sliding short. The following paper presents an analysis of a phase shifter consisting of two ganged shifters of the Magid type in tandem; that is, a differential phase shifter, as proposed by Beatty.

A desirable characteristic of the differential phase shifter when compared to the Magid type is a reduction of the error that is introduced by short circuit displacement measurement tolerances. This paper contains graphical data for WR-90 and WR-112 waveguides so it can readily be determined which type of phase shifter has the least limit of error for a given measurement. A comparison example will be included.

The sources of error which are considered include those introduced by reflectometer tuning imperfections, waveguide width tolerances, short circuit displacement measurement and short circuit misalignment errors. Only these errors are considered because they limit the over-all accuracy that can be attained with either phase shifter.

THEORY

Magid[1] proposed a phase shifter consisting of a directional coupler, matching transformers and precision waveguide section terminated in a sliding short circuit, as shown in Fig. 1. Assuming that $\Gamma_{2i}=0$ and $S_{31}=0$, the change of phase of the emerging signal b_3 is exactly equal

Manuscript received February 3, 1964; revised April 9, 1964.
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[1] M. Magid, "Precision microwave phase shift measurements," IRE TRANS. ON INSTRUMENTATION, vol. I-7, pp. 321–331; December, 1958.

to the change of phase of the load attached to the second arm. If the load is a sliding short circuit, the output signal can be varied in phase by changing the relative position of the short circuit at arm 2. Ideally, the change of phase of this type phase shifter, at a single frequency, is

$$\psi_T = \frac{720(l_1 - l_2)}{\lambda_{g1}} \text{ degrees} \tag{1}$$

where $(l_1 - l_2)$ is the distance the short was moved and λ_{g1} is the waveguide wavelength. (The significance of the double subscript will be explained in a later paragraph.)

Schafer and Beatty[2] performed an error analysis on this type of phase shifter and presented graphically the limits of error associated with tuning imperfections and dimensional tolerances. It can be concluded from this analysis that the accuracy of this type of phase shifter is limited mainly by the errors introduced through the measurement of waveguide width and short circuit position.

The error introduced by the short circuit positional tolerance can be reduced through the use of the differential phase shifter proposed by Beatty.[3] An arrangement is shown in Fig. 2. Basically, it consists of a ganged pair of phase shifters of the type shown in Fig. 1 connected in series; however, the precision waveguide sizes differ, resulting in different waveguide wavelengths.

Assume that the short circuit combination is moving upward as indicated in Fig. 2. The phase of the emerging signal from Phase Shifter 1 is then advanced with respect to its input and can, in the ideal case, be computed from the equation

$$\psi_1 = \frac{720(l_1 - l_2)}{\lambda_{g1}} \text{ degrees} \tag{2}$$

where $(l_1 - l_2)$ is the distance the short is moved within the waveguide having guide wavelength λ_{g1}. At the same time the emerging signal from Phase Shifter 2 is retarded with respect to its input by the amount

$$\psi_2 = \frac{720(l_1 - l_2)}{\lambda_{g2}} \text{ degrees} \tag{3}$$

where λ_{g2} is the guide wavelength in the precision section of Phase Shifter 2.

Because the output from Phase Shifter 1 is the incident signal to Phase Shifter 2, the total phase shift of the output signal with respect to the input will then be the sum of (2) and (3);

$$\psi_T = \frac{720(l_1 - l_2)}{\lambda_{g1}}\left(1 - \frac{\lambda_{g1}}{\lambda_{g2}}\right). \tag{4}$$

A comparison of (1) and (4) shows that, for an equal movement $(l_1 - l_2)$ of the short circuits, the total phase shift of the ganged system shown in Fig. 2 differs from that of the system of Fig. 1 by the factor

$$\left(1 - \frac{\lambda_{g1}}{\lambda_{g2}}\right),$$

λ_{g1} being the same in both figures. It can be seen that a differential phase shift occurs when

$$\frac{\lambda_{g1}}{\lambda_{g2}} < 2.$$

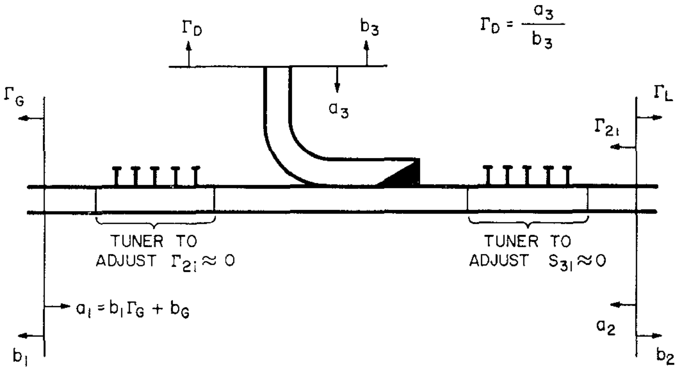

Fig. 1—A standard microwave phase shifter.

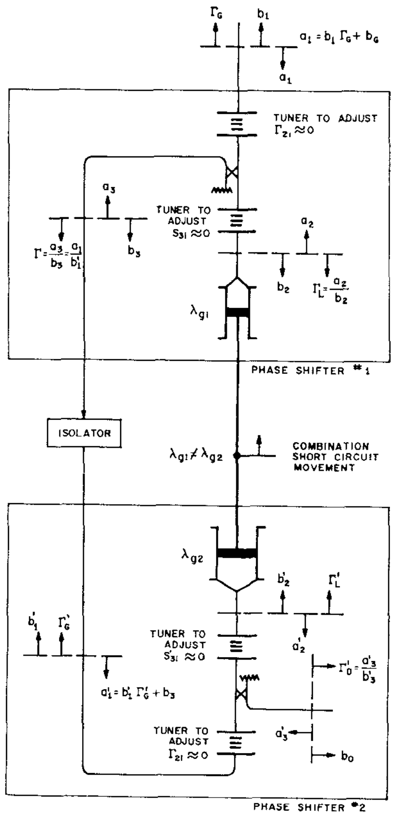

Fig. 2—A differential phase shifter.

[2] G. E. Schafer and R. W. Beatty, "Error analysis of a standard microwave phase shifter," *J. Res. NBS*, vol. 64C, pp. 261–265; October–December, 1960.

[3] R. W. Beatty, "A differential microwave phase shifter," IEEE Trans. on Microwave Theory and Techniques (*Correspondence*), vol. MTT-12, pp. 250–251; March, 1964.

Tuning Errors

It can be shown[4] that the output of the phase shifter shown in Fig. 1 can be expressed in the form

$$\frac{b_3}{b_G} = \frac{\begin{vmatrix} S_{21} & S_{22} \\ S_{31} & S_{32} \end{vmatrix} \Gamma_L + S_{31}}{\begin{vmatrix} (1 - S_{11}\Gamma_G) & S_{13}\Gamma_D \\ S_{31}\Gamma_D & (1 - S_{33}\Gamma_D) \end{vmatrix} (1 - \Gamma_{2i}\Gamma_L)} \tag{5}$$

where the S_{mn} terms are the scattering coefficients of the junction, Γ_L, Γ_G and Γ_D are the reflection coefficients of the sliding load, generator, and detector, respectively, and Γ_{2i} is the reflection coefficient as seen from the load.

If $S_{31} = \Gamma_{2i} = 0$, and the magnitude of Γ_L remains constant, the ideal change of phase of the output signal for a change of length Δl is

$$\Delta\psi_3 = {}^f\psi_L - {}^i\psi_L \tag{6}$$

where the superscripts designate the initial and final phases of the precision waveguide termination. Departures from this ideal response result from the inability to adjust $S_{31} = 0$ and $\Gamma_{2i} = 0$. The error introduced by each of these has been analyzed by Schafer and Beatty.[3]

The output of the first phase shifter in Fig. 2, when operated alone, can be expressed by (5). The output of the second phase shifter, in Fig. 2, when operated alone can then be expressed as

$$\frac{b_0}{b_3} = \frac{\begin{vmatrix} S_{21}' & S_{22}' \\ S_{31}' & S_{32}' \end{vmatrix} \Gamma_L' + S_{31}'}{\begin{vmatrix} (1 - S_{11}'\Gamma_G') & S_{13}'\Gamma_D' \\ (S_{31}'\Gamma_G') & (1 - S_{33}'\Gamma_D') \end{vmatrix} (1 - \Gamma_{2i}'\Gamma_L')}. \tag{7}$$

Inspection of Fig. 2 reveals that, when the two phase shifters are cascaded without isolation, the quantity Γ_D of (5) is a function of the magnitude and angle of the reflection coefficient of the "short circuit" terminating arm 2 of the second coupler. Also, the quantity Γ_G' in (7) is a function of the magnitude and angle of the reflection coefficient of the "short circuit" terminating arm 2 of the first coupler. Therefore, to facilitate this analysis, infinite isolation is assumed between the two phase shifters when connected in series.

Utilization of this assumption allows the signal output to be expressed in terms of its input by use of the equation

$$\frac{b_0}{b_G} = \frac{b_0}{b_3} \times \frac{b_3}{b_G}, \tag{8}$$

b_3 being the output wave of Phase Shifter 1 and the input wave of Phase Shifter 2.

[4] R. W. Beatty, and D. M. Kerns, "Recently developed microwave impedance standards and methods of measurement," IRE Trans. on Instrumentation, vol. I-7, pp. 319–321; December, 1958.

The total tuning error results from the inability to tune completely for the conditions $\Gamma_{2i} = 0$, $\Gamma_{2i}' = 0$, $S_{31} = 0$ and $S_{31}' = 0$. The error contributed by failure to achieve each of these conditions will be considered, and the various errors will be totaled to determine the overall limit of error.

Case I

$$S_{31} = S_{31}' = 0; \qquad \Gamma_{2i} \neq 0; \qquad \Gamma_{2i} \neq 0.$$

The phase variation ϵ for this case, as derived from (5), (7), and (8), is

$$\epsilon = \text{argument of} \frac{(1 - \Gamma_{2i}{}^i\Gamma_L)}{(1 - \Gamma_{2i}{}^f\Gamma_L)} + \frac{(1 - \Gamma_{2i}'{}^i\Gamma_L')}{(1 - \Gamma_{2i}'{}^f\Gamma_L')} \tag{9}$$

where ϵ is the variation in change of phase of $b_0{}^i/b_0{}^f$, as compared to change of phase of $\Gamma_L{}^i/\Gamma_L{}^f$. Expression (9) provides for the calculation of the error if the complex quantities Γ_{2i} and Γ_{2i}' are known. We shall proceed to evaluate the maximum error resulting from measured maximum magnitude deviations of these complex quantities.

The limit of error of the first term of (9) was shown by Schafer and Beatty[3] to be

$$\lim \epsilon \approx 2|\Gamma_{2i}|\left|\sin\frac{\psi_1}{2}\right|. \tag{10}$$

In a like manner, the magnitude of the second term in (9) will then be

$$\lim \epsilon' \approx 2|\Gamma_{2i}'|\left|\sin\frac{\psi_2}{2}\right|. \tag{11}$$

The total limit of error is then given by adding (10) and (11).

$$\lim \epsilon_T = 2|\Gamma_{2i}|\left|\sin\frac{\psi_1}{2}\right| + 2|\Gamma_{2i}'|\left|\sin\frac{\psi_2}{2}\right|. \tag{12}$$

Fig. 3 is a graph for determining the values of $|\Gamma_{2i}|$ and $|\Gamma_{2i}'|$ when the reflectometer tuning procedure outlined by Engen and Beatty[5] is utilized. The final output variation achieved when arm 2 is terminated in a highly reflective, phaseable load is represented on the X axis. The limit of error of the differential system can be computed from (12), using the values of $|\Gamma_{2i}|$ and $|\Gamma_{2i}'|$ from Fig. 3 and the values of ψ_1 and ψ_2 determined from (2) and (3).

In an actual system, it is probable that the reflectometer will be tuned for the same final output variation, which results in the condition $|\Gamma_{2i}| = |\Gamma_{2i}'|$. When this happens (12) reduces to

[5] G. F. Engen and R. W. Beatty, "Microwave reflectometer techniques," IRE Trans. on Microwave Theory and Techniques, vol. MTT-7, pp. 351–355; July, 1959.

$$\lim \epsilon_T = 2 |\Gamma_{2i}| \left\{ \left| \sin \frac{\psi_T}{2\left(1 - \frac{\lambda_{g1}}{\lambda_{g2}}\right)} \right| + \left| \sin \frac{\left(\psi_T \frac{\lambda_{g1}}{\lambda_{g2}}\right)}{2\left(1 - \frac{\lambda_{g1}}{\lambda_{g2}}\right)} \right| \right\} \tag{13}$$

when ψ_T is the total change of phase of the unknown as determined from (4). The value for $|\Gamma_{2i}|$ is again that given in Fig. 3. In Fig. 4 are plotted values of the function

$$K = \left\{ \left| \sin \frac{\psi_T}{2\left(1 - \frac{\lambda_{g1}}{\lambda_{g2}}\right)} \right| + \left| \sin \frac{\psi_T \frac{\lambda_{g1}}{\lambda_{g2}}}{2\left(1 - \frac{\lambda_{g1}}{\lambda_{g2}}\right)} \right| \right\} \tag{13a}$$

when WR-90 and WR-112 waveguides are used in Phase Shifters 1 and 2, respectively.

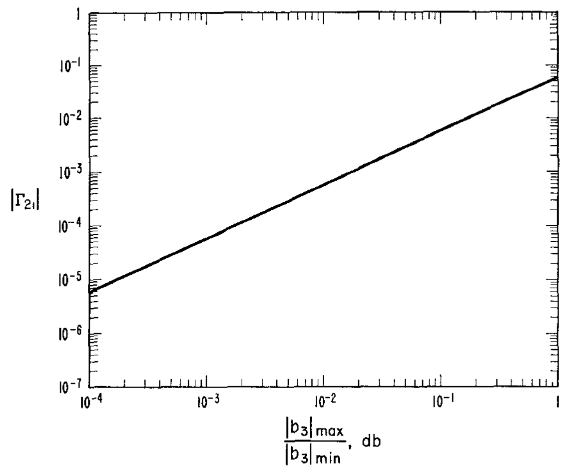

Fig. 3—Graph for determination of $|\Gamma_{2i}|$.

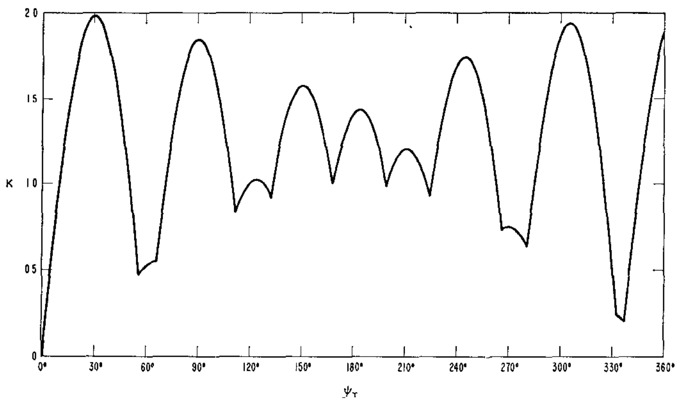

Fig. 4—Tuning error factor.

CASE II

$$\Gamma_{2i} = \Gamma_{2i}' = 0, \quad \text{but} \quad S_{31} \neq 0, \qquad S_{31}' \neq 0.$$

The phase variation ϵ for this case, as derived from (5), (7) and (8) is

$$\epsilon = \text{argument of} \left\{ \frac{1 + \dfrac{S_{31}}{(S_{32}S_{21} - S_{31}S_{22})^f \Gamma_L}}{1 + \dfrac{S_{31}}{(S_{32}S_{21} - S_{31}S_{22})^i \Gamma_L}} \right\} + \text{argument of} \left\{ \frac{1 + \dfrac{S_{31}'}{(S_{32}'S_{21}' - S_{31}'S_{22}')^f \Gamma_L'}}{1 + \dfrac{S_{31}'}{(S_{32}'S_{21}' - S_{31}'S_{22}')^i \Gamma_L'}} \right\}. \tag{14}$$

The error caused by the first portion of the expression was shown by Schafer and Beatty[2] to be

$$\lim \epsilon \approx 2 \left| \frac{S_{31}}{S_{32}S_{21}} \right| \sin \left| \frac{\psi_1}{2} \right|. \tag{15}$$

The error due to the second portion of (14) can be shown to be

$$\lim \epsilon' \approx 2 \left| \frac{S_{31}'}{S_{32}'S_{21}'} \right| \sin \left| \frac{\psi_2}{2} \right|, \tag{16}$$

resulting in a total error of

$$\lim \epsilon_T \approx 2 \left| \frac{S_{31}}{S_{32}S_{21}} \right| \left| \sin \frac{\psi_1}{2} \right| + 2 \left| \frac{S_{31}'}{S_{32}'S_{21}'} \right| \left| \sin \frac{\psi_2}{2} \right|. \tag{17}$$

Fig. 5 is a graph for determining values of

$$\left| \frac{S_{31}'}{S_{32}'S_{21}'} \right| \quad \text{and} \quad \left| \frac{S_{31}}{S_{32}S_{21}} \right|$$

when the reflectometer tuning procedure outlined in Engen and Beatty[5] is utilized. The final output variation achieved when arm 2 is terminated in a low reflection, phaseable load is represented on the X axis.

In an actual system, it is probable that the reflectometers will be tuned for the same final output variation, which results in the condition

$$\left| \frac{S_{31}}{S_{32}S_{21}} \right| = \left| \frac{S_{31}'}{S_{32}'S_{21}'} \right|.$$

When this occurs, (17) reduces to

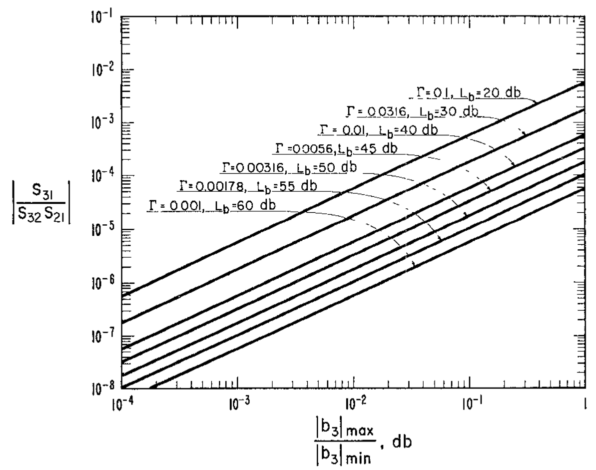

Fig. 5—Graph for determination of $|S_{31}/S_{32}S_{21}|$.

$$\lim \epsilon_T \approx 2\left|\frac{S_{31}}{S_{32}S_{21}}\right|\left[\left|\sin\frac{\psi_T}{2\left(1-\dfrac{\lambda_{g1}}{\lambda_{g2}}\right)}\right| + \left|\sin\frac{\left(\psi_T\dfrac{\lambda_{g1}}{\lambda_{g2}}\right)}{2\left(1-\dfrac{\lambda_{g1}}{\lambda_{g2}}\right)}\right|\right]. \quad (18)$$

Figs. 4 and 5 can now be used to determine the limit of error for WR-90 and WR-112 waveguides. On Fig. 5, L_b is the return loss of the termination used for tuning. The corresponding reflection coefficient for the termination Γ is also shown.

Dimensional Errors

The phase shift of the ideal device was given by (4) as

$$\psi_T = \frac{720(l_1 - l_2)}{\lambda_{g1}}\left(1 - \frac{\lambda_{g1}}{\lambda_{g2}}\right).$$

The phase error caused by uncertainty in measurement of the axial motion of the sliding short circuit combination can be found by considering the differential of the above expression with respect to l_1 and l_2.

$$\epsilon_l = \frac{\partial\psi_T}{\partial l_1}dl_1 + \frac{\partial\psi_T}{\partial l_2}dl_2 = \frac{720}{\lambda_{g1}}\left(1 - \frac{\lambda_{g1}}{\lambda_{g2}}\right)(dl_1 - dl_2). \quad (19)$$

The quantities dl_1 and dl_2 are the tolerances associated with the measurement of l_1 and l_2. Therefore, for a given system, $|dl_1| = |dl_2| = |\Delta l|$, and the maximum error occurs when these tolerances are added.

$$\lim \epsilon_l = \frac{1440}{\lambda_{g1}}\left(1 - \frac{\lambda_{g1}}{\lambda_{g2}}\right)|\Delta l|. \quad (20)$$

Fig. 6 is a graph for determining the error for the WR-90, WR-112 combination.

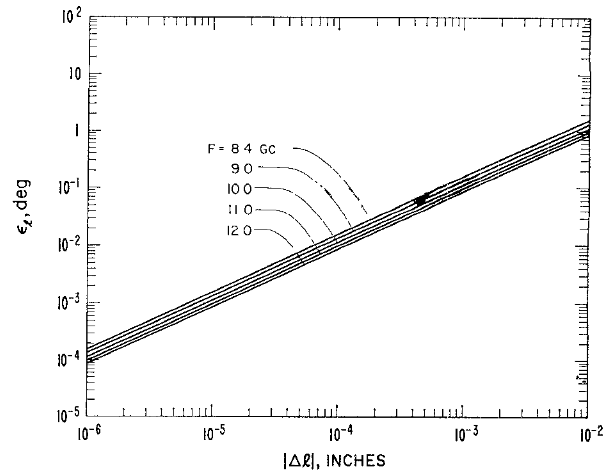

Fig. 6—Limit of motional error for WR-90 and WR-112.

The error due to nonuniformity of both waveguides can be found by determining the differential of (4) with respect to a, the waveguide width.

$$\epsilon_a = \frac{\partial(\psi_T)}{\partial\lambda_{g1}}\frac{\partial\lambda_{g1}}{\partial a_1}da_1 + \frac{\partial(\psi_T)}{\partial\lambda_{g2}}\frac{\partial\lambda_{g2}}{\partial a_2}da_2. \quad (21)$$

$$\lim \epsilon_a = \left[\frac{\lambda_{g1}}{4a_1^3}|\Delta a_1| + \frac{\lambda_{g2}}{4a_2^3}|\Delta a_2|\right]\cdot(l_1 - l_2)720. \quad (22)$$

As was done in Shafer and Beatty,[2] the limit of this error is presented as a fractional error.

$$\lim \frac{\epsilon_a}{\psi_T} = \frac{\lambda_{g1}^2}{4a_1^3\left(1 - \dfrac{\lambda_{g1}}{\lambda_{g2}}\right)}|\Delta a_1| + \frac{\lambda_{g1}\lambda_{g2}}{4a_2^3\left(1 - \dfrac{\lambda_{g1}}{\lambda_{g2}}\right)}|\Delta a_2| = \left|\frac{\epsilon_{a1}}{\psi_T}\right| + \left|\frac{\epsilon a_2}{\psi_T}\right|. \quad (23)$$

The values for $|\epsilon_{a1}/\psi_T|$ and $|\epsilon_{a2}/\psi_T|$ can be determined from Fig. 7 for each phase shifter where the use of WR-90 and WR-112 is assumed in the respective phase shifters.

The final error to be evaluated is introduced through the misalignment of the short circuit combination in respect to the displacement measurement instrument, as shown in Fig. 8.

The measured change of phase is a function of the length l,

$$\psi_T \text{ measured} = \frac{720l}{\lambda_{g1}}\left(1 - \frac{\lambda_{g1}}{\lambda_{g2}}\right). \quad (24)$$

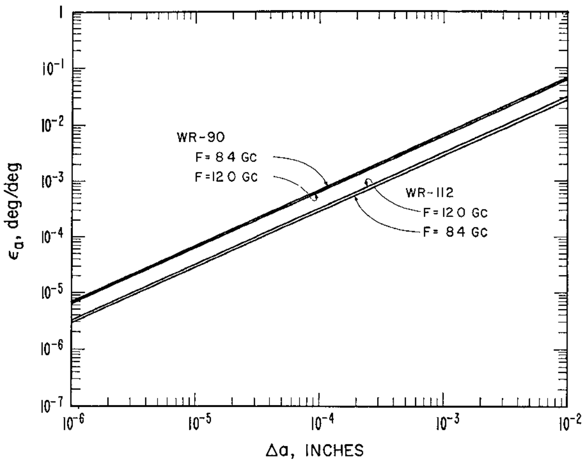

Fig. 7—Limit of tolerance error for WR-90 and WR-112.

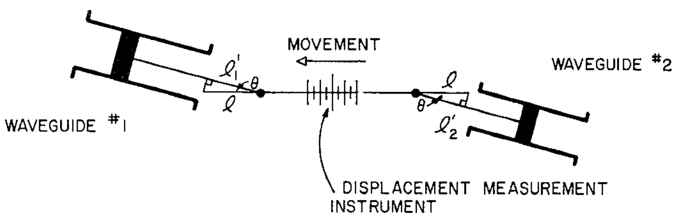

Fig. 8—Precision waveguide misalignment.

The actual change of phase is a function of the dimensions l_1' and l_2',

$$l_1' = l \cos \theta_1 \tag{25a}$$

$$l_2' = l \cos \theta_2. \tag{25b}$$

The actual change of phase of the system can be determined with the following equation:

$$\psi_T \text{ actual} = 720\left(\frac{l_1'}{\lambda_{g1}} - \frac{l_2'}{\lambda_{g2}}\right). \tag{26}$$

The total phase error introduced into the measurement as a result of waveguide misalignment is found by subtracting (26) from (24).

$$\epsilon_T = \frac{720}{\lambda_{g1}}(l_1' - l) - \frac{720}{\lambda_{g2}}(l_2' - l). \tag{27}$$

The use of (25a) and (25b) and the series expansion for $\cos \theta$ through the second order reduces (27) to

$$\epsilon = \frac{720l}{\lambda_{g2}}\left(\frac{\theta_2^2}{2}\right) - \frac{720l}{\lambda_{g1}}\left(\frac{\theta_1^2}{2}\right) \tag{28}$$

where θ_1 and θ_2 represent the small angular misalignments of the short circuits with respect to the measurement device during the course of measurement.

Eq. (28) may be utilized for calculation of the total error resulting from the misalignment when actual angles are known and are constant. In the practical sense, however, waveguide alignment tolerances are used to determine the angles θ_1 and θ_2. Therefore, it is possible for the alignment errors to add or subtract, resulting in either an error accumulation or error difference. The maximum error will occur when the two add.

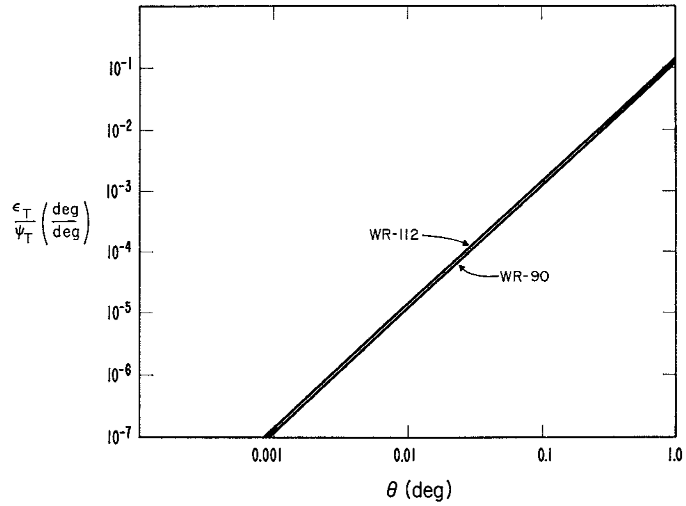

Fig. 9—Limit of misalignment error for WR-90 and WR-112.

$$\text{limit } \epsilon_T = \left|\frac{720l}{\lambda_{g2}}\left(\frac{\theta_2^2}{2}\right)\right| + \left|\frac{720l}{\lambda_{g1}}\left(\frac{\theta_1^2}{2}\right)\right|. \tag{29}$$

The angles θ_1 and θ_2 are now the maximum deviation from boresight. When presented as a fractional error (29) becomes

$$\text{limit } \frac{\epsilon_T}{\psi_T} = \left|\frac{\theta_1^2}{2\left(1 - \frac{\lambda_{g1}}{\lambda_{g2}}\right)}\right| + \left|\frac{\theta_2^2 \frac{\lambda_{g1}}{\lambda_{g2}}}{2\left(1 - \frac{\lambda_{g1}}{\lambda_{g2}}\right)}\right|; \tag{30}$$

the values for each term in (30) can be determined from Fig. 9 and combined for the over-all error. The use of WR-90 and WR-112 is assumed in the respective phase shifters.

As an example of the use of the data presented here, assume a differential phase shifter was made and used as follows. The load attached to arm 2 of each reflectometer is made with a short circuit adjustable to 0.0005-inch maximum uncertainty. The tolerance associated with both waveguides is ± 0.0001 inch. The tuning procedure for Γ_{2i} and Γ_{2i}' was carried out to a 0.01-db variation in the maximum to minimum response for each. The tuning for S_{31} and S_{31}' was carried out to a 1.0-db variation in the maximum to minimum response for each with a tuning load that has a return loss of 45 db. Assume the short circuit misalignment is a maximum of 0.1°. The change of phase is 60° and the operating frequency is 9 gigahertz.

From Fig. 3, $|\Gamma_{2i}|$ for a 0.01-db variation is 0.00058. From Fig. 4, K for a measured angle of 60° is 0.517. From (13), the limit error is 0.0006 radian or 0.034°. From Fig. 5, $|S_{31}/S_{32}S_{21}|$ for a 1-db variation with a return loss of 45 db is 0.00032. From (18), the limit of error is 0.00033 radian or 0.019°. From Fig. 6, for a positional tolerance of 0.0005 inch, the limit of motional error is 0.066°. From Fig. 7, the errors associated with the waveguide tolerances are 0.00068° per degree for WR-90 and 0.00032 for WR-112. For 60°, there is a limit of waveguide dimensional error of 0.06°. From Fig. 9, for a 0.1° short circuit misalignment the total phase error is 0.0014° per degree for WR-90 and 0.0012° per degree for WR-119. For 60°, this is a limit of error of 0.156°. The total limit of error from all these sources is then 0.335°. If a single phase shifter was built to these specifications in WR-90, the total error would be 0.437°.

If, however, for the same system specifications the measured angle was 90°, the total error for the differential system would be 0.585° as compared to 0.464° for the single system. In this example, the reduction of positional error is more than offset by an increase in tuning error.

Conclusions

Although the instrument described in this paper provides for a smaller short circuit positional error as compared with that for a single phase shifter, the combined tuning and waveguide dimensional errors are increased. For certain measured angles, the over-all increase of error will more than offset the positional error reduction achieved through the use of a differential system.

This instrument should be particularly useful where small changes of phase angle are involved because all the errors, other than the short circuit positional error, are proportional to the measured angle.

Acknowledgment

The author extends appreciation to W. A. Downing for computing and graphically presenting the data in Fig. 4.

Maximum Bandwidth Performance of a Nondegenerate Parametric Amplifier with Single-Tuned Idler Circuit

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Summary—**The single-tuned bandwidth and limiting flat bandwidth of a nondegenerate reflection-type diode parametric amplifier is calculated. The amplifier has a broad-banding filter structure in the signal circuit and a single-tuned idler circuit. An experimental low-noise, wide-band *L*-band amplifier is described, and measurement results are presented. The amplifier has a triple-tuned signal circuit and a single-tuned idler circuit and is pumped at 11.3 Gc. A nearly flat bandwidth of 23 per cent at 7 db gain and an effective input noise temperature of 70°K at room temperature ambient and of 29°K at liquid nitrogen (77°K) ambient has been obtained.**

I. Introduction

Several authors have discussed the design and gain-bandwidth limitation of a nondegenerate diode parametric amplifier of the reflection type employing broad-banding filters in signal and idler circuit.[1-3] It has not been established, however, whether it is necessary to have broad band-pass filters in both circuits. In this paper the gain-bandwidth limitation of a similar amplifier employing a band-pass filter in the signal circuit only (and a single-tuned filter in the idler circuit) is investigated theoretically and experimentally, for the following two reasons. 1) For low microwave frequency operation, the idler frequency can be made so much higher than the signal frequency that the absolute bandwidth of the idler circuit is much broader than that

Manuscript received March 16, 1964; revised April 20, 1964. This paper was submitted posthumously. Correspondence concerning it should be addressed to R. S. Engelbrecht, Bell Telephone Laboratories, Inc., Murray Hill, N. J.

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